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Relatively Prime Domination Number in Quadrilateral Snake Graphs

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Abstract:

A set $S \subseteq V$ is said to be relatively prime dominating set if it is a dominating set with at least two elements and for every pair of vertices u and v in S, $(\deg(u), \deg(v)) = 1$. The minimum cardinality of a relatively prime dominating set is called relatively prime domination number and it is denoted by $\gamma_{rpd}(G)$. If there is no such pair exist, then $\gamma_{rpd}(G) = 0$. For a finite undirected graph $G(V, E)$ and a subset $\sigma \subseteq V$, the switching of G by σ is defined as the graph $G^{\sigma}(V, E')$ which is obtained from G by removing all edges between σ and its complement V-σ and adding as edges all non-edges between σ and V-σ. This article delves into the discussion of the relatively prime domination number on quadrilateral snake graphs and their complements. The findings reveal that for quadrilateral snake graphs, the relatively prime domination number $\gamma_{rpd}(G^v)$ equals either 2, 3 or 4. Similarly, for alternate quadrilateral snake graphs, the $\gamma_{rpd}(G^v)$ is determined to be 2, 3 or 4. In the case of double quadrilateral snake graphs, the relatively prime domination number $\gamma_{rpd}(G^v)$ is established as 2, 3, 4, 6 or 7, while for double alternate quadrilateral snake graphs, it is 2, 3, 4 or 5. Notably, the complements of quadrilateral, alternate quadrilateral, double quadrilateral, and double alternate quadrilateral snake graphs exhibit a relatively prime domination number of 2.

Keywords: Dominating Set, Domination Number, Relatively Prime Dominating Set, Relatively Prime Dominating Number

1. Introduction

By a graph G = (V, E) we mean a finite undirected graph without loops and multiple edges. The order and size of G are denoted by p and q respectively. For graph theoretical terms, we refer to Harary [2] and for terms related to domination we refer to Haynes [7]. A subset S of V is said to be a dominating set in G if every vertex in V– S is adjacent to at least one vertex in S. The domination number γ(G) is the minimum cardinality of a dominating set in G. Berge [1] and Ore [6] formulated the concept of domination in graphs. It was further extended to define many other dominations related parameters in graphs. In 2017, C. Jayasekaran and A. Jancy Vini [3] have introduced the concept of relatively prime domination number in graph theory. Let G be a non – trivial graph. A set $S \subseteq V$ is said to be a relatively prime dominating set if it is a dominating set and for every pair of vertices u and v in S such that (d(u), d(v)) = 1. The minimum cardinality of a relatively prime dominating set is called the relatively prime domination number and it is denoted by $\gamma_{rpd}(G)$. Further they have introduced the concept of relatively prime dominating polynomial in [4]. Switching in graphs was introduced by Lint and Seidel [5]. For a finite undirected graph G(V, E) and a subset $\sigma \subseteq$V, the switching of G by σ is defined as the graph

$G^{\sigma}(V, E')$ which is obtained from G by removing all edges between σ and its complement $V-\sigma$ and adding as edges all non-edges between σ and $V-\sigma$. For σ = {v}, we write G^v instead of $G^{\{v\}}$ and the corresponding switching is called as vertex switching. In this paper we determine the relatively prime domination number $\gamma_{rpd}(G^v)$ and $\gamma_{rpd}(\overline{G})$, where G is a quadrilateral snake graph.

2. Preliminaries

Definition 2.1. A **quadrilateral snake** is obtained from a path a_1,a_2, …,a_n by joining a_i and a_{i+1} to new vertices b_i and c_i respectively and joining the vertices b_i and c_i for i = 1,2, …, n – 1. That is every edge is of a path is replaced by a cycle C_4.

Definition 2.2. An **alternate quadrilateral snake** is obtained from a path a_1,a_2, …,$a_n$by joining a_i and a_{i+1} to new vertices b_i and c_i respectively and joining the vertices b_i and c_i for $i \equiv 1(\text{mod } 2)$ and i ≤ n – 1 and then joining b_i and c_i. That is every alternate edge of a path is replaced by a cycle C_4. It is denoted by $A(Q_n)$.

Definition 2.3. A **double quadrilateral snake** is obtained from two quadrilateral snakes that have a common path. It is denoted by $D(Q_n)$.

Definition 2.4. An **alternate double quadrilateral snake** is obtained from two alternative quadrilateral snakes that have a common path. It is denoted by $A(D(Q_n))$.

3. Relatively Prime Domination Number of Quadrilateral Snake Graph

In this section we have discussed the relatively prime domination number for snake graphs.

Theorem 3.1. Let G be a quadrilateral snake graph with p vertices, where p = 3n+1, n ≥ 2. Then $\gamma_{rpd}(G^v)$ = 2, 3 or 4.

Proof: Let G be a quadrilateral snake graph with p vertices. Let the vertices in the path be v_1, v_2, …, v_p and the vertices in the quadrilateral be $u_1, u_2, w_2, u_3, w_3, \ldots, u_{m-1}, w_{m-1}, u_m, w_m$. Then the degree of vertices in the path except the initial and the end vertex is 4; the degree of initial and the end vertex is 2; the degree of vertices in the quadrilateral is 2. Let v be a vertex in G. We have the following cases.

Case 1: v is any vertex from $\{u_1, u_2, w_2, u_3, w_3, \ldots, u_{m-1}, w_{m-1}, u_m\}$.

Without loss of generality, let v = u_i, i = 1, 2, …, m-1. Then d(u_i) = p–3. Clearly, this vertex covers all the vertices except two vertices, say w_{i-1} and v_i. Then d(w_{i-1}) = 1 and d(v_i) = 1 if v_i is an initial(end) vertex, otherwise d(v_i) = 3. To cover the vertex w_{i-1} and v_i, either we have to take these two vertices or take a vertex which is adjacent to both w_{i-1} and v_i. Such a vertex always will exist, since it is a quadrilateral graph and |V| ≥ 6. Let the vertex be v_t. Then d(v_t) = 5 if it is an internal path vertex; otherwise d(v_t) = 3. We have two more subcases.

Case 1.1: v_i is an initial(end) vertex.

If d(v) is not a multiple of 5, then $\{v, v_t\}$ is a relatively prime dominating set. Hence $\gamma_{rpd}(G^v)$ = 2 in this case. If d(v) is multiple of 5, then the set $\{v, w_{i-1}, v_i\}$ is our required relatively prime dominating set. Therefore, $\gamma_{rpd}(G^v)$ = 3 in this case.

Case 1.2: v_i is not an initial(end) vertex.

Then d(v_t) = 3. Since |V| = 3n+1 and d(v) = p–3, the degree of v cannot be a multiple of 3 and so (p–3, 3) = 1. Thus, the set $\{v, v_t\}$ is our required relatively prime dominating set. Therefore, $\gamma_{rpd}(G^v)$ = 2 in this case.

Case 2: v is an initial or an end vertex of a path.

Without loss of generality, let it be v_1. Then d(v) = p-3. This vertex does not cover the two vertices u_1 and v_2. Then d(v_2) = 3 and d(u_1) = 1. To cover the vertices u_1 and v_2, two possibilities are there. Either we have to take these two vertices or a vertex which is adjacent to both u_1 and v_2. Such a vertex always exists, since G is a quadrilateral snake graph and |V| ≥ 6. Then the vertex must be u_2 and d(u_2) = 3. Since p–3 is not a multiple of 3, we have the set $\{v,\ u_2\}$ is our required relatively prime dominating set and hence $\gamma_{rpd}(G^v)$ = 2.

Case 3: v is any internal path vertex.

Without loss of generality, let it be v_i. Then d(v_i) = p-5. This vertex does not cover four vertices; namely, v_{i-1}, v_{i+1}, u_i and w_i. Then d(u_i) = d(w_i) = 1 and d(v_{i-1}) = 1 if it is an initial vertex and d(v_{i+1}) = 3; similarly d(v_{i-1}) = 3 and d(v_{i+1}) = 1 if it is an end vertex. To cover these four vertices, either we have to take these four vertices or the vertices which are adjacent to these four vertices. Since G is a quadrilateral graph, such a vertex always exists as in Case 2. Let them be w_{i-1} and u_{i+2} and degree of these two vertices is three and hence we cannot take these two vertices together. Suppose v_{i-1} is an initial vertex, then the set $\{v_i, v_{i-1}, u_i, u_{i+2}\}$ is a relatively prime dominating set and hence $\gamma_{rpd}(G^v)$ = 4. Similarly, if v_{i+1} is an end vertex. Hence assume that neither v_{i-1} is an initial vertex nor v_{i+1} is an end vertex. Since we cannot take the vertices w_{i-1} and u_{i+2} together, we have to choose a vertex v_{i-1} or v_{i+1}. But both of them has degree 3. Therefore, relatively prime dominating set does not exist in this case.

Theorem 3.2. Let G be an alternate quadrilateral snake graph with p vertices, where p = 4n, n ≥ 2. Then $\gamma_{rpd}(G^v)$ = 2, 3 or 4.

Proof: Let G be an alternate quadrilateral snake graph with p vertices. Let the vertices in the path be v_1,v_2,v_6,…,v_m and the vertices in the quadrilateral be u_1 ,u_2 ,u_3,…,u_m. Then degree of each vertex in the path except the initial and end vertex is 3; degree of initial and end vertex is 2; degree of vertices in the quadrilateral is 2. Let v be any vertex in G**.** We have the following cases:

Case 1: v is any vertex from $\{u_1, u_2, \ldots, u_m\}$.

Without loss of generality, we take v = $u_i,\ i = 1,2,\ldots,m$. Then d(v) = p-3. This vertex covers all the vertices of G^v, except the two vertices, namely v_i and u_{i-1}. Then d(v_i) =1 if v_i is initial or end vertex, otherwise 2. To cover the vertices $v_i$and u_{i-1},either we have to take these two vertices or choose a vertex which is adjacent to both v_i and u_{i-1}. Such a vertex always exists in alternate quadrilateral snake graph. Let the vertex be v_{i+1} and d(v_{i+1}) = 4. Since d(v_i) = p-3 and $|V|$ = 4n , it cannot be multiple of 4 and hence (d(v_i), d(v_{i+1})) = (p-3, 4) =1 and these two vertices covers all the vertices of G^v. Hence relatively prime dominating set is $\{u_i, v_{i+1}\}$ and $\gamma_{rpd}(G^v)$ = 2.

Case 2: v is an initial vertex or an end vertex.

Without loss of generality, let v = v_1. Then d(v) = p-3. This vertex covers all the vertices except two vertices, namely u_1 and v_2 and d(u_1) =1 and d(v_2) = 2. To cover the vertices u_1 and v_2, either we have to choose these two vertices or a vertex which is adjacent to both u_1 of v_2. Such a vertex is always existing, since G is an alternate quadrilateral snake graph. Let the vertex be u_2 and d(u_2) = 3. If d(v) is not a multiple of 3, the set {v,u_2} satisfies all the condition for being a relatively prime dominating set. Hence γ_{rpd}= 2 in this case. Suppose that d(v) is a multiple of 3. Since $|V|$ = 4n , p-3 is always odd. Hence {v, u_1 ,v_2} is a relatively prime dominating set. Thus γ_{rpd}= 3 in this case.

Case 3: v is any internal path vertex.

Let it be v_i. Then d(v_i) = p-4. This vertex covers all the vertices of G^v, except the 3 vertices, namely v_{i-1}, v_{i+1}, u_i and d(u_i) = 1; d(v_{i-1}) = 1 if v_{i-1} is an initial vertex, otherwise 2. Similarly d(v_{i+1}) = 1 if v_{i+1} is an initial vertex, otherwise 2. Since G is an alternate quadrilateral snake graph, let the vertex adjacent to u_i and v_{i-1} be u_{i-1} and the vertex adjacent to v_{i+1} be u_{i+1} and v_{i+2} and d(u_{i-1}) = d(u_{i+1}) = 3; d(v_{i+2}) = 4. Since d(v) is a multiple of 4, we cannot take the vertex v_{i+2}. We have the following subcases.

Case 3.1: v_{i-1} is an initial vertex and v_{i-1} is not an end vertex.

If d(v) is not a multiple of 3, then the set $\{v_i, u_i, v_{i-1}, u_{i+1}\}$ is a relatively prime dominating set and hence $\gamma_{rpd}(G^v)$ = 4. If d(v) is a multiple of 3, then relatively prime dominating set does not exist.

Case 3.2: v_{i+1} is an end vertex and v_{i-1} is not an initial vertex.

Same as Case 3.1.

Case 3.3: Neither v_{i-1} is an initial vertex nor v_{i+1} is an end vertex.

Then d(v_{i-1}) = d(v_{i+1}) = 2. Therefore, we cannot choose these two vertices. Also note that the vertices which are adjacent to v_{i-1} and v_{i+1} of degree 3 and 4. Therefore, relatively prime dominating set does not exist.

Theorem 3.3. Let G be a double quadrilateral snake graph with p vertices, where p = 5n+1, n $\geq$ 2. Then $\gamma_{rpd}(G^v)$ = 2, 3, 4, 6 or 7.

Proof: Let G be a double quadrilateral snake graph with p vertices. Let the vertices in the path be $v_1, v_2, \dots, v_m$ and the vertices in the upper quadrilateral be $u_1, u_2, w_2, u_3, w_3, \dots, u_{m-1}, w_{m-1}, u_m$ and the vertices in the lower quadrilateral be $x_1, x_2, y_2, x_3, y_3, \dots, x_{m-1}, y_{m-1}, x_m$. Then degree of each internal vertex is 6; degree of initial and end vertex is 3; degree of vertices in the upper and lower quadrilateral is 2. Let v be any vertex in G. We consider the following cases:

Case 1: v is any vertex from $\{u_1, u_2, w_2, u_3, w_3, \dots, u_{m-1}, w_{m-1}, u_m,$

$x_1, x_2, y_2, x_3, y_3, \dots, x_{m-1}, y_{m-1}, x_m\}$.

Without loss of generality, let v = u_i, $i = 1,2, \dots, m$. Then d(v) = p-3 in G^v. This vertex u_i covers all the vertices in G^v other than the two vertices which are adjacent to u_i in G, namely v_i, w_{i-1}. Since G is a quadrilateral snake graph, the vertices v_i and w_{i-1} are adjacent with a vertex v_{i-1}. To cover the vertices v_i and w_{i-1}, either we have to take these two vertices or which is adjacent to both v_i and w_{i-1}, that is, the vertex v_{i-1}. Note that if n is even and odd, then d(v) is odd and even respectively. Here, d(w_{i-1}) = 1; d(v_i) = 2 if v_i is a initial vertex or end vertex, otherwise 5. And d(v_{i-1}) = 7 if v_{i-1} is an internal path vertex, otherwise 4. We have the following subcases.

Case 1.1: v_i is an initial or an end vertex.

Then d(v_i) = 2. If d(v) is odd and not a multiple of 7, then $\{v, v_{i-1}\}$ is a relatively prime dominating set and hence $\gamma_{rpd}(G^v)$ = 2 in this case. If d(v) is odd and a multiple of 7, then the set $\{v, v_i, w_{i-1}\}$ is a relatively prime dominating set and hence $\gamma_{rpd}(G^v)$ = 3 in this case. If d(v) is even and not multiple of 7, then the set $\{v, v_{i-1}\}$ is a relatively prime dominating set and hence $\gamma_{rpd}(G^v)$ = 2. If d(v) is even and multiple of 7, then relatively prime dominating set does not exist in this case.

Case 1.2: v_i is neither an initial nor an end vertex.

Then d(v_i) = 5. Since d(v) = p-3 and |V| = 5n+1, degree of v cannot be a multiple of 5. Suppose that v_{i-1} is an initial or end vertex. Then d(v_{i-1}) = 4. If d(v) is odd, then the set $\{v, v_{i-1}\}$ is a relatively prime dominating set and hence $\gamma_{rpd}(G^v)$ = 2. If d(v) is even and not a multiple of 4, then the set $\{v, v_{i-1}\}$ is a relatively prime dominating set and hence $\gamma_{rpd}(G^v)$ = 2. If d(v) is even and multiple of 4, then the set $\{v, v_i, w_{i-1}\}$ is a relatively prime dominating set and hence $\gamma_{rpd}(G^v)$ = 3.

Case 2: v is an initial vertex or an end vertex.

Without loss of generality, let v = v_1. Then d(v) = p-4 in G^v. Then the vertex v_1 covers all the vertices except the three vertices, namely, u_1, x_1 and v_2 and d(u_1) = d(x_1) = 1; d(v_2) = 5. If n is even and odd, then d(v) = p-4 is even and odd respectively. To cover the vertices u_1, x_1 and v_2, either we have to choose these vertices or a vertex which are adjacent to u_1, x_1 and v_2. Note that there is no vertex which is adjacent to these three vertices, since G is a double quadrilateral snake graph. But u_1, v_2 and x_1, v_2 are connected by a vertex. They are u_2 and x_2 and d(u_2) = d(x_2) = 3. Note that, since |V| = 5n+1 and d(v) = p-4, degree of v cannot be a multiple of 5. If d(v) is odd and not a multiple of 3, then the set $\{v, u_2, x_1\}$ is a relatively prime dominating set and hence $\gamma_{rpd}(G^v)$ = 3. If d(v) is odd and a multiple of 3, then the set $\{v, u_1, x_1, v_2\}$ is a relatively prime dominating set and hence $\gamma_{rpd}(G^v)$ = 4. If d(v) is even and not multiple of 3, then the set $\{v, u_2, x_1\}$ is a relatively prime dominating set and hence $\gamma_{rpd}(G^v)$ = 3. If d(v) is even and multiple of 3, then the set $\{v, u_1, x_1, v_2\}$ is a relatively prime dominating set and hence $\gamma_{rpd}(G^v)$ = 4.

Case 3: v is any internal path vertex.

Without loss of generality, let it be v_i, $i = 2,3, \dots, m-1$. Then d(v_i) = p-7 in G^v. Then the vertex v_i covers all the vertices except the six vertices, namely $u_i, w_i, x_i, y_i, v_{i-1}, v_{i+1}$. Then d($u_i$) = d($w_i$) = d($x_i$) = d($y_i$) = 1 and d($v_{i-1}$) = 2 if it is an initial vertex, otherwise 5. Similarly for the vertex v_{i+1}. To cover these six vertices, either we have to take these six vertices or vertices which are adjacent to these six vertices. As in case 2, the vertices which are adjacent to u_i and v_{i-1}, w_i and v_{i+1}, x_i and v_{i-1}, y_i and v_{i+1} are $w_{i-1}, u_{i+1}, x_{i-1}, y_{i+1}$ respectively. Then d(w_{i-1}) = d(u_{i+1}) = d(x_{i-1}) = d(y_{i+1}) = 3. To obtain a relatively prime dominating set, we can take only one of these four vertices. We have the following subcases.

Case 3.1: v_{i-1} is an initial vertex.

Then d(v_{i-1}) = 2. Note that d(v) cannot be a multiple of 5. If d(v) is odd and not a multiple of 3, then the set $\{v, u_i, x_i, v_{i-1}, u_{i+1}, y_i\}$ is a relatively prime dominating set and hence $\gamma_{rpd}(G^v)$ = 6. If d(v) is odd and a multiple of 3, then the set $\{v, u_i, x_i, w_i, y_i, v_{i+1}, v_{i-1}\}$ is a relatively prime dominating set and hence $\gamma_{rpd}(G^v)$ = 7.

Case 3.2: v_{i+1} is an end vertex

Same as Case 3.1.

Case 3.3: v_{i-1} is an initial vertex and v_{i+1} is an end vertex.

Then |V| must be 11 and hence d(v) =4. Hence the set $\{v, u_i, x_i, v_{i-1}, u_{i+1}, y_i\}$ is a relatively prime dominating set and hence $\gamma_{rpd}(G^v)$ = 6.

Case 3.4: Neither v_{i-1} is an initial vertex nor v_{i+1} is an end vertex.

Then d(v_{i-1}) = d(v_{i+1}) = 5. Since degree of these two vertices are 5, we cannot take these vertices together. So we consider the vertices which are adjacent to v_{i+1}, namely $u_{i+1}, x_{i+1}, w_{i+1}, y_{i+1}, v_{i+2}$. Then d($u_{i+1}$) = d($x_{i+1}$) = d($w_{i+1}$) = d($y_{i+1}$) = 3 and d($v_{i+2}$) = 4 if v_{i+2} is an end vertex, otherwise 7. If d(v) is odd and not a multiple of 3, then the set $\{v, u_i, x_i, v_{i-1}, u_{i+1}, y_i\}$ is a relatively prime dominating set and hence $\gamma_{rpd}(G^v)$ = 6. Suppose that d(v) is odd and a multiple 3. Here we cannot choose a vertex of degree 3. The only possibility is choose the vertex v_{i+2}. If v_{i+2} is an end vertex, then the set $\{v, u_i, x_i, v_{i-1}, w_i, y_i, v_{i+2}\}$ is a relatively prime dominating set and hence $\gamma_{rpd}(G^v)$ = 7. If v_{i+2} is not an end vertex and d(v) is not a multiple of 7, then set $\{v, u_i, x_i, v_{i-1}, w_i, y_i, v_{i+2}\}$ is a relatively prime dominating set and hence $\gamma_{rpd}(G^v)$ = 7. If d(v) is odd and a multiple of 3 and 7, then relatively prime dominating set does not exist. If d(v) is even and not a multiple of 3, then the set $\{v, u_i, x_i, v_{i-1}, w_i, u_{i+1}\}$ is a relatively prime dominating set and hence $\gamma_{rpd}(G^v)$ = 6. Suppose that d(v) is even and a multiple of 3. As said above, we cannot take the vertices of degree 3. We can cover the vertices $u_i, w_i, v_{i-1}, x_i, y_i$. We have only one vertex to cover is v_{i+1}. If v_{i+2} is an end vertex, then the set $\{v, u_i, x_i, v_{i-1}, w_i, y_i, v_{i+2}\}$ is a relatively prime dominating set and hence $\gamma_{rpd}(G^v)$ = 7. If v_{i+2} is not an end vertex and d(v) is not a multiple of 7, then set $\{v, u_i, x_i, v_{i-1}, w_i, y_i, v_{i+2}\}$ is a relatively prime dominating set and hence $\gamma_{rpd}(G^v)$ = 7. If d(v) is odd and a multiple of 3 and 7, then relatively prime dominating set does not exist.

Theorem 3.4. Let G be a double alternate quadrilateral snake graph with p vertices, where p = 6n, n ≥ 2. Then $\gamma_{rpd}(G^v)$ = 2, 3, 4 or 5.

Proof: Let G be a double alternate quadrilateral snake graph with p vertices. Let the vertices in the path be v_1,v_2, …,v_m, where v_1 and v_m denote the initial and end vertex respectively. Let the vertices in the upper quadrilateral be u_1,u_2,…, u_m and the vertices in the lower quadrilateral be w_1,w_2,…,w_m. Then degree of each internal path vertex is 4; degree of initial and end vertex is 3; degree of vertices in the quadrilateral is 2. Let v be any vertex in G. We have the following cases.

Case 1 : v is any vertex from $\{u_1, u_2, \ldots, u_{m-1}, w_1, w_2, \ldots, w_{m-1}\}$.

Without loss of generality, let v = $u_i, i = 1,2, \ldots, m$. Then d(v) = p–3. This vertex covers all the vertices in G^v except the two vertices, namely v_i and u_{i+1} or v_i and u_{i-i}. Without loss of generality, let us take v_i and u_{i+1}. To find the relatively prime dominating set, we have to cover these two vertices. Either we have to choose these two vertices or a vertex which is adjacent to both the vertices v_i and v_{i+1}. Such a vertex always exists, since G is a double alternative quadrilateral snake graph and let the vertex be v_{i+1}. Then d(v_{i+1}) = 4, if v_{i+1} is an end vertex, otherwise 4. Since |V| = 6n, d(v) is always odd and it is a multiple of 3. We consider the following subcases.

Case 1.1: v_i is an initial vertex.

Then the set $\{v, v_i, u_{i+1}\}$ is a relatively prime dominating set, since (p-3, 4) = 1. Therefore, $\gamma_{rpd}(G^v)$ = 3.

Case 1.2: v_i is not an initial vertex.

Consider the vertex v_{i+1}. If v_{i+1} is an end vertex, then the set $\{v, v_{i+1}\}$ is a relatively prime dominating set and hence . Otherwise, we have degree of v_{i+1} is 5. If d(v) is not a multiple of 5, then the set $\{v, v_{i+1}\}$ is a relatively prime dominating set and hence $\gamma_{rpd}(G^v)$ = 2. If d(v) is not multiple of 5, then relatively prime dominating set does not exist.

Case 2: v is an initial vertex or an end vertex.

Without loss of generality, let v = v_1. Then d(v_1) = p–4. This vertex covers all the vertices of G^v except three vertices, namely u_1, w_1, and v_2. Note that there is no vertex which covers all these three vertices. But the vertices u_1 and v_2, w_1 and v_2 are connected by a vertex, namely u_2 and w_2. Then d(u_2) = d(w_2) = 3. Since d(v_1) = p-4, it cannot be multiple of 3. Hence the set $\{v_1, u_2, w_1\}$ is a relatively prime dominating set and hence $\gamma_{rpd}(G^v)$ = 3.

Case 3: v is anyone of internal path vertex.

Without loss of generality, let v = $v_i, i = 2,3, \ldots, m-1$. Then d(v) = p–5. This vertex covers all the vertices of G^v except four vertices, namely, u_i, w_i, v_{i-1} and v_{i+1}. Then d(u_i) = d(w_i) = 1, d(v_{i-1}) = 2 if v_{i-1} is an initial vertex, otherwise 3. Similarly, d(v_{i+1}) = 2 if v_{i+1} is an end vertex, otherwise 3. We consider the following subcases.

Case 3.1: v_{i-1} is an initial vertex.

Note that d(v) = p-5 is always odd and not multiple of 3. As said in case 2, the vertex which is adjacent to u_i and v_{i-1} is u_{i-1}, is of degree 3 and the vertex adjacent to the vertex v_{i+1} in the paths is v_{i+2}, is of degree 5, if v_{i+2} is not an end vertex, otherwise 4. If v_{i+2} is an end vertex, then the set $\{v_i, v_{i+2}, u_{i-1}, w_i\}$ is a relatively prime dominating set and hence $\gamma_{rpd}(G^v)$ = 4. If v_{i+2} is not an end vertex and d(v) is not a multiple of 5, then the set $\{v_i, v_{i+2}, u_{i-1}, w_i\}$ is a relatively prime dominating set and hence $\gamma_{rpd}(G^v)$ = 4. If v_{i+2} is not an end vertex and d(v) is a multiple of 5, then the set $\{v_i, u_i, w_i, v_{i-1}, v_{i+1}\}$ is a relatively prime dominating set and hence $\gamma_{rpd}(G^v)$ = 5.

Case 3.2:

Same as Case 3.1.

Case 3.3: Neither v_{i-1} is an initial vertex nor v_{i+1} is an end vertex.

Then d(v_{i-1}) = d(v_{i+1}) = 3. So, we cannot take these two vertices together. Consider the vertices adjacent to v_{i-1} and v_{i+1} in the path. Let them be v_{i-2} and v_{i+2}. Then d(v_{i-2}) = 4 if it is an initial vertex, otherwise 5. Then the set $\{v_i, v_{i-2}, u_{i+1}, w_i\}$ is a relatively prime dominating set if v_{i-2} is an initial vertex and hence $\gamma_{rpd}(G^v)$ = 4. If v_{i-2} is not an initial vertex, v_{i+2} is not an end vertex and d(v) is not a multiple of 5, then the set $\{v_i, v_{i-2}, u_{i+1}, w_i\}$ is a relatively prime dominating set and hence $\gamma_{rpd}(G^v)$ = 4. Otherwise, relatively prime dominating set does not exist.

4. Relatively Prime Domination Number on Complement of Quadrilateral Snake Graph

In this section we have shown that the relatively prime domination number for complement of quadrilateral type graphs is 2.

Theorem 4.1. Let G be a quadrilateral snake graph with p vertices. Then for p is even, $\gamma_{rpd}(\bar{G})$ = 2.

Proof: Let G be a quadrilateral snake graph with p vertices. Let the vertices be v_1,v_2, … v_p. Since degree of each vertex in the quadrilateral snake graph G is either 2 or 4, degree of each vertex in the complement of quadrilateral graph $\bar{G}$ is either p–3 or p–5. Note that if either n is even or odd, then p–3 and p–5 are always odd and hence (p–3, p–5) = 1. Consider a vertex which has degree p–3. Then this vertex, say v_i covers all the vertices of $\bar{G}$ except the vertices, say v_k and v_l. In order to find a relatively prime dominating set, choose a vertex of degree p–4, say v_j which has adjacency with the vertices v_k and v_l. Note that such a vertex is always exist, since |V| ≥ 6. Since these two vertices v_i and v_j satisfies the conditions for being a relatively prime dominating set, we have $\gamma_{rpd}(\bar{G})$ = 2.

Theorem 4.2. For any alternate quadrilateral snake graph G, $\gamma_{rpd}(\bar{G})$ = 2

Proof: Let G be alternate quadrilateral snake graph with p vertices .Let the vertices in the path be v_1,v_2,… ,v_p. Since degree of each vertex in an alternate quadrilateral snake graph is either 2 or 3, degree of each vertex in the complement of alternate quadrilateral snake graph is either p-3 or p-4. Choose a vertex of degree p-3, say v_i. This vertex cover all the vertices of $\bar{G}$ except two vertices, namely v_k and v_l. Now, choose a vertex of degree p-4, say v_j such that it has adjacent with the vertices v_k and v_l. Such a vertex always exists, since$|V| \geq 6$. Since these two vertices v_i and v_j satisfies the conditions for being a relatively prime dominating set, we have $\gamma_{rpd}(\bar{G})$ = 2.

Theorem 4.3. For any double quadrilateral snake graph G, $\gamma_{rpd}(\bar{G})$ = 2.

Proof: Let G be a double quadrilateral snake graph with p vertices. Let the vertices be v_1,v_2, … v_p. We know that, in the double quadrilateral snake graph, degree of each vertex is either 2,3 or 6. Hence in the complement graph $\bar{G}$, degree of each vertex is either p–3, p–4 or p–7. Choose a vertex of degree p–3, say v_i. Since this vertex cover all the vertices of $\bar{G}$ except two vertices say, v_k and v_l, we have to choose a vertex of degree n – 4 such that it has adjacency with the two vertices v_k and v_l. Such a vertex always exists, since |V| ≥ 6. Let the vertex which has degree p–4 be v_j. Now, clearly the two vertices v_i and v_j cover all the vertices of $\bar{G}$ and (d(v_i),d(v_j)) = (p–3, p–4) = 1. Therefore, relatively prime dominating set is $\{v_i, v_j\}$ and hence $\gamma_{rpd}(\bar{G})$ = 2.

Theorem 4.4. For any double alternate quadrilateral snake graph G, $\gamma_{rpd}(\bar{G})$ = 2.

Proof: Let G be a double alternate quadrilateral snake graph with p vertices. Let the vertices in G be $v_1, v_2, \dots, v_p$. Then degree of each vertex in the complement of double alternative quadrilateral snake graph $\bar{G}$ is either p-3 , p-4 or p-5, since degree of each vertex in a double alternate quadrilateral snake graph is 2, 3 or 4. Consider a vertex which has degree p-3, say v_i. This vertex covers all the vertices of $\bar{G}$ except two vertices, say v_k and v_l. Now, choose a vertex degree p-4 such that it has adjacency with the vertices v_k and v_l . Such a vertex is always possible, since $|V| > 6$. Let the vertex which has degree p-4 be v_j. Hence the relatively prime dominating set is $\{v_i, v_j\}$ and the relatively prime domination number is 2.

5. Conclusion

Dominations in graph theory is a wide area with more applications to real life which helps the researchers to get more ideas to manage the problems in real life situation. The standard purpose of the paper is to explain the significance of dominating sets and relatively prime domination number. We have examined the idea of relatively prime dominations in various types of quadrilateral snake graphs and also their complements.

References

[1] Berge C. 1962. Theory of Graphs and its Applications. London: Methuen.

[2] Harary F. 1972. Graph Theory. California: Addison-Wesley Publishing company Reading, Massachusetts.

[3] Jayasekaran C., and Jancy Vini A. 2017. Relatively Prime Dominating sets in Graphs. Annals of Pure and Applied Mathematics. https://dx.doi.org/10.22457/apam.v14n3a2.

[4] Jayasekaran C., and Jancy Vini A. 2019. Relatively Prime Dominating Polynomial in Graphs. Malaya Journal of Matematik. https://doi.org/ 10.26637/MJM0704/0006

[5] Lint J. H., and Seidal J. J. 1966. Equilateral Points in Elliptic Geometry. In Proc. Kon. Nede. Acad. Wetensch. Ser. A, Vol. 69, pp. 335-348.

[6] Ore O. 1962. Theory of Graphs. American Mathematical Society Colloquium Publications, vol. 38 (Amer. Math. Soc., Providence, RI).

[7] Teresa W. Haynes, Stephen Hedetniemi, and Peter Slater. 1998. Fundamentals of domination in graphs. New York: Marcel Dekker, Inc..

[8] Anat Jaslin Jini, A, Jancy Vini A, Manikanda Prabhu S, Suresh K and Chellamani P, “Relatively Prime Domination Number in Quadrilateral Snake Graphs”, Advances in Nonlinear Variational Inequalities. Vol 28, No. 2 (2025).